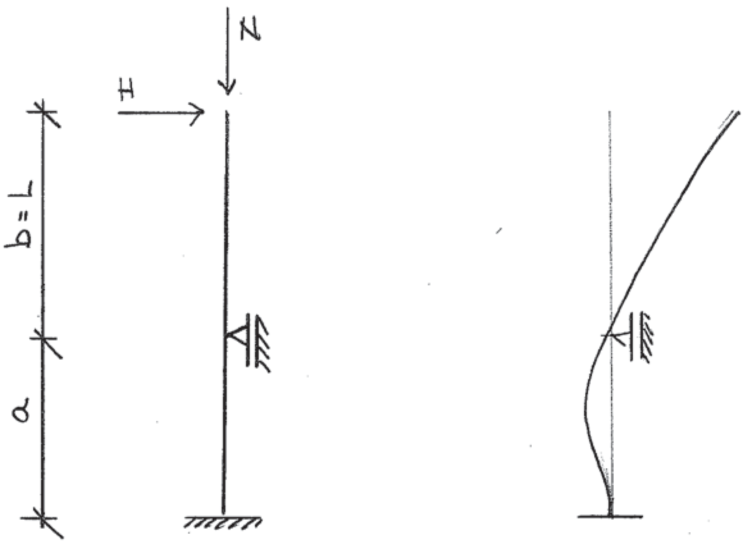
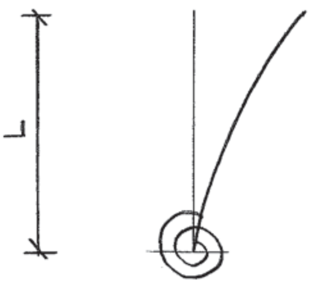
EXAMPLE

Braced 1st floor
unbraced 2nd floor



What is the buckling lenght of 2nd floor ?

EN 1992-1-1 figure 5.7.



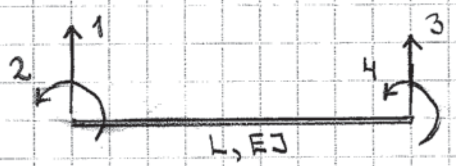
$L_0 > 2L$

$$(5.16) \quad l_0 = l \cdot \max\left\{\sqrt{1+10\frac{k_1 k_2}{k_1+k_2}}\;;\;\left(1+\frac{k_1}{1+k_1}\right)\left(1+\frac{k_2}{1+k_2}\right)\right\}$$

Now $k_2 = \infty$ no restraint at all
$k_1 = ?$

$$k = \frac{\theta}{M}\,\frac{EJ}{l}$$

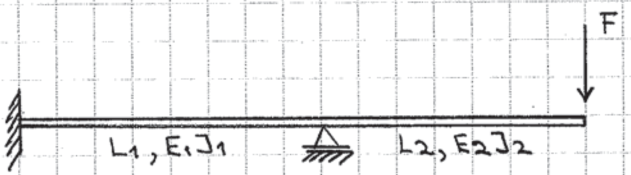
Beam FEM



$$[k]\{d\} = \{F\}$$

$$[k] = \frac{EJ}{L^3}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{symm} & & & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

Now structure



Beam 1: $$[k^1] = \frac{E_1J_1}{L_1^3}\begin{array}{c} 1 \\ \begin{bmatrix} 4L_1^2 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \end{array}$$

Beam 2: $$[k^2] = \frac{E_2J_2}{L_2^3}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 4L_2^2 & -6L_2 & 2L_2^2 \\ -6L_2 & 12 & -6L_2 \\ 2L_2^2 & -6L_2 & 4L_2^2 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}$$

$$\{d\} = \begin{Bmatrix} \varphi_1 \\ v_2 \\ \varphi_3 \end{Bmatrix} \quad , \quad \{F\} = \begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}$$

$$[K] = [k^1] + [k^2]$$

$$L_1 = L_2 = L$$

$$E_1 J_1 = E_2 J_2 = EJ$$

$$\frac{EJ}{L^3} \begin{bmatrix} 8L^2 & -6L & 2L^2 \\ -6L & 12 & -6L \\ 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \varphi_1 \\ v_2 \\ \varphi_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}$$

eq(1): $\cancel{\frac{EJ}{L^3}} \left(8L^2\varphi_1 - 6L v_2 + 2L^2\varphi_3 \right) = 0$

$$6 v_2 = 8L\varphi_1 + 2L\varphi_3$$

$$v_2 = \frac{4}{3}L\varphi_1 + \frac{1}{3}L\varphi_3$$

eq(3): $2L^2\varphi_1 - 6L v_2 + 4L^2\varphi_3 = 0 \quad | : L$

$$2L\varphi_1 - \frac{6 \cdot 4}{3}L\varphi_1 - \frac{6}{3}L\varphi_3 + 4L\varphi_3 = 0 \quad | : L$$

$$2\varphi_1 - 8\varphi_1 - 2\varphi_3 + 4\varphi_3 = 0$$

$$-6\varphi_1 + 2\varphi_3 = 0 \quad \Rightarrow \quad \varphi_3 = 3\varphi_1$$

$$\Rightarrow v_2 = \frac{4}{3}L\varphi_1 + \frac{3}{3}L\varphi_1 = \frac{7}{3}L\varphi_1$$

eq(2) $\frac{EJ}{L^3}\left(-6L\varphi_1 + 12 v_2 - 6L\varphi_3 \right) = F$

$$-6L\varphi_1 + \frac{12 \cdot 7}{3}L\varphi_1 - 6 \cdot 3 \cdot L\varphi_1 = \frac{FL^3}{EJ}$$

$$4\varphi_1 = \frac{FL^2}{EJ}$$

EN 1992-1-1 (5.16)

$$k = \frac{\vartheta}{M} \frac{EJ}{L}$$

$$\vartheta \approx \varphi_1$$
$$M = FL$$

$$\Rightarrow 4EJ\varphi_1 = FL^2$$

$$\Rightarrow \frac{\varphi_1}{FL} \frac{EJ}{L} = \frac{1}{4}$$

$$\Rightarrow k_1 = \frac{1}{4} = 0{,}25$$
$$k_2 = \infty$$

$$(5.16) \quad l_0 = l \cdot \max\left\{\sqrt{1 + 10\frac{k_1 k_2}{k_1 + k_2}} \; ; \; \left(1 + \frac{k_1}{1+k_1}\right)\left(1 + \frac{k_2}{1+k_2}\right)\right\}$$

$$= l \max\{1{,}87 \; ; \; 2{,}4\}$$

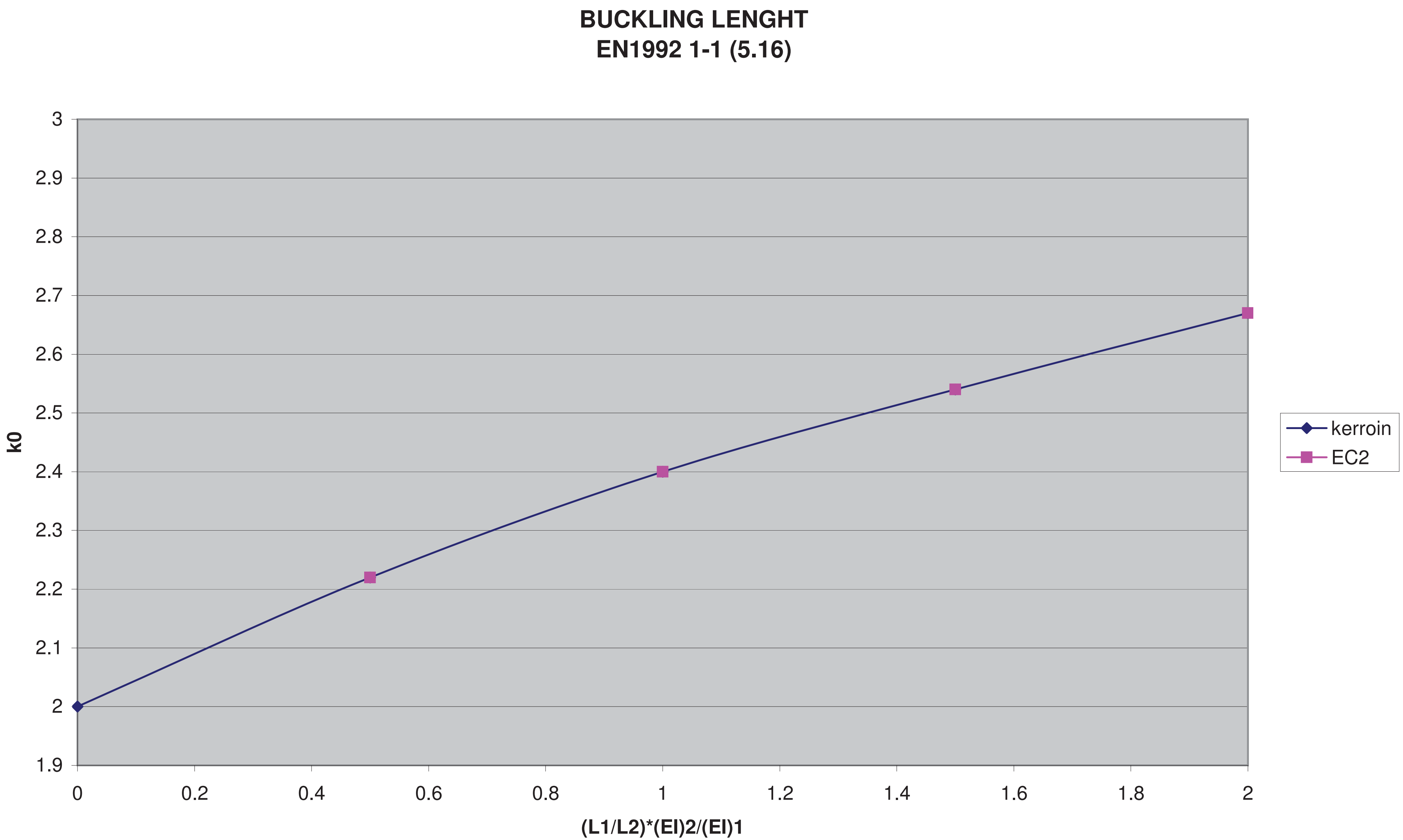
$$\boxed{\Rightarrow l_0 = 2{,}4\, l}$$

a/b	k_0
0	2,0
0,5	2,22
1,0	2,40
1,5	2,54
2,0	2,67

Stiffness matrix for $[k^1]$

$$[k^1] = \frac{E_1 J_1}{l_1^3} \cdot 4 l_1^2 = 4 \frac{E_1 J_1}{l_1}$$

$$\Rightarrow \quad \frac{\frac{E_1 J_1}{l_1}}{\frac{E_2 J_2}{l_2}} = \frac{l_2}{l_1} \frac{E_1 J_1}{E_2 J_2}$$



BUCKLING LENGHT
EN1992 1-1 (5.16)
k0
3
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2
1.9
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
(L1/L2)*(EI)2/(EI)1
kerroin
EC2